General announcements

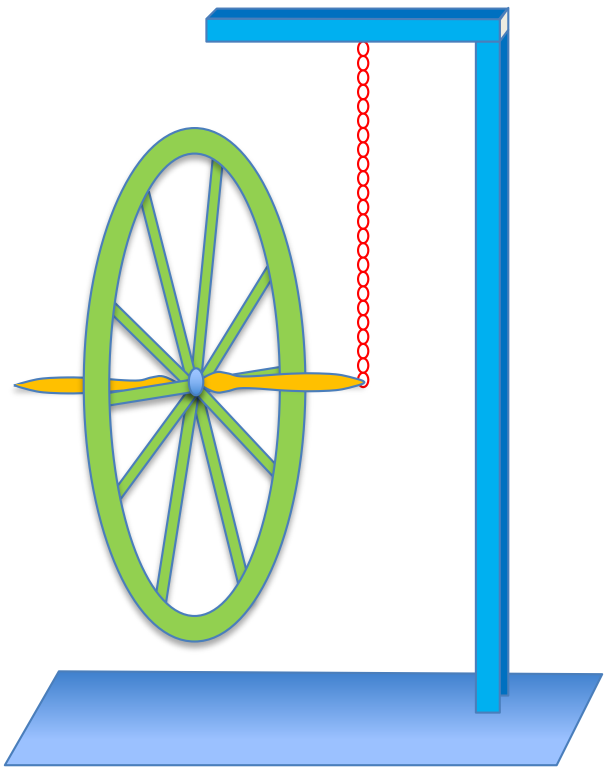
- Done with quizzes - now you have a sense of how you are with the fundamentals of rotational motion—talk about what's ahead . . .
  - If there's a section you feel shaky on, come see me before block days start to go over it!

- Today: the wheel and pulsars, then talking about the block days, including:
  - Study resources for the rotational motion unit test
  - Goalless problem guidelines and practice

- Tomorrow/Monday: looking through the possible rotational motion test questions + working on goalless practice

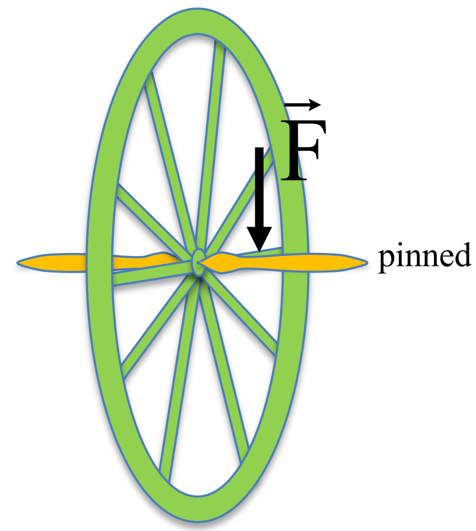
- Tuesday 3-4ish next week, I will be available in Poly 109 to answer questions

Back to the Non-AP Tom-Foolery



Remember the precessing wheel demonstration?

It was accompanied by a second demonstration in which a torque was quickly applied to the pinned axle of a rotating wheel, and instead of the wheel following the direction of the applied force, the wheel jerked to the right or left, depending upon the direction of the wheel's rotation.



It's time to make sense of both of these.

The mathematical key to these seemingly mysterious behaviors are wrapped up in the relationship between torque and angular momentum (or, Newton's Second Law, rotation style). That is, the relationship:

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$

This relationship suggests one of two things.

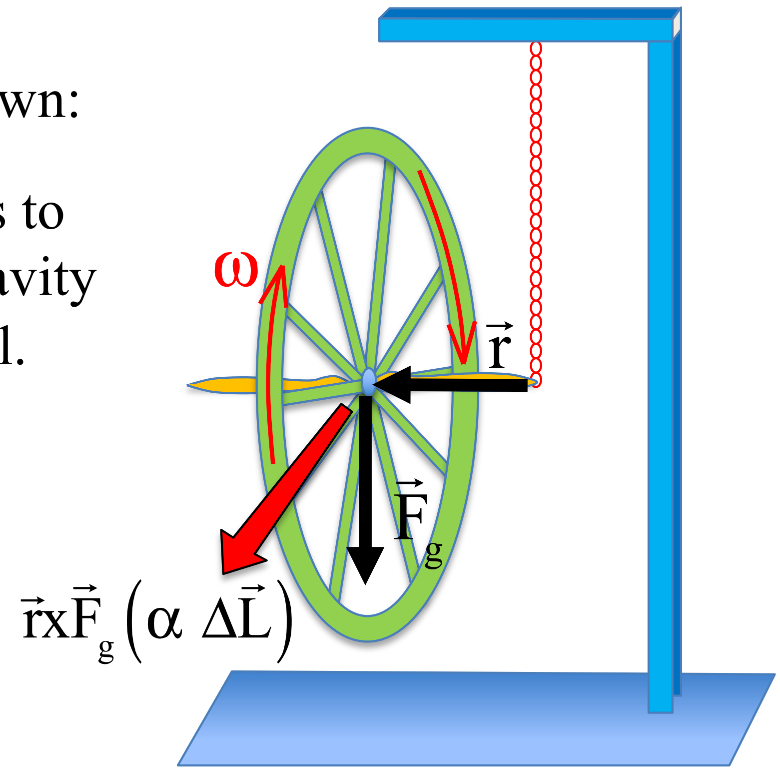
1.) If the direction of the net torque applied to a body matches the direction of the body's angular momentum vector (translation: it matches the direction of the body's angular velocity), an applied torque will change the magnitude of the angular momentum (that is, the body will angularly speed up or slow down). The translational parallel to this is a force along the line of motion making a body speed up or slow down.

But:

2.) If the direction of the net torque applied to a body does not match the direction of a body's angular momentum vector, the body's angular momentum will still change but the change will be in the angular momentum's DIRECTION, not its magnitude. The translational parallel to this is a force that is perpendicular to the line of motion creating a centripetal situation. The precessing wheel circumstance falls into this latter category.

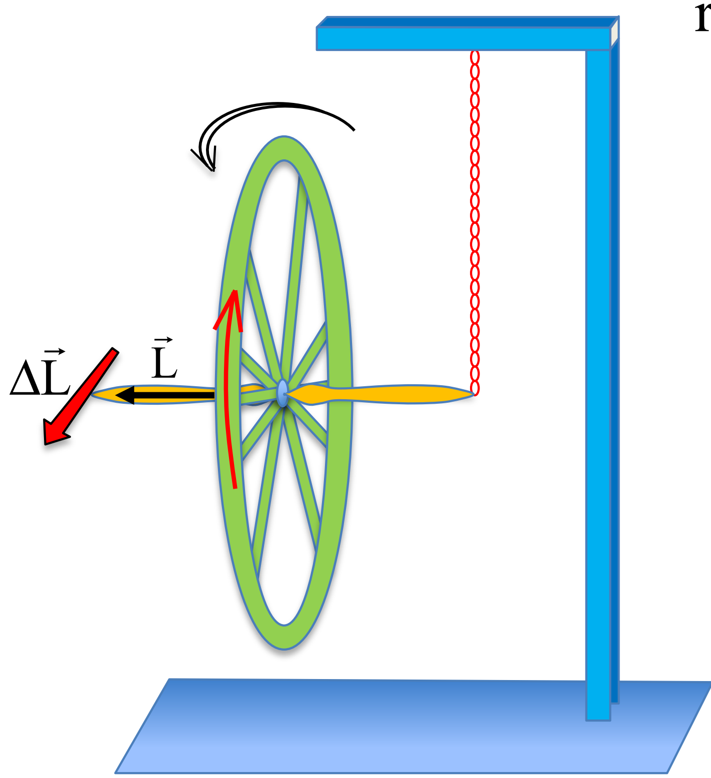
Starting with the hanging wheel rotating as shown:

The system is pinned where the chain attaches to the axle. The force being applied is due to gravity and happens at the center of mass of the wheel.

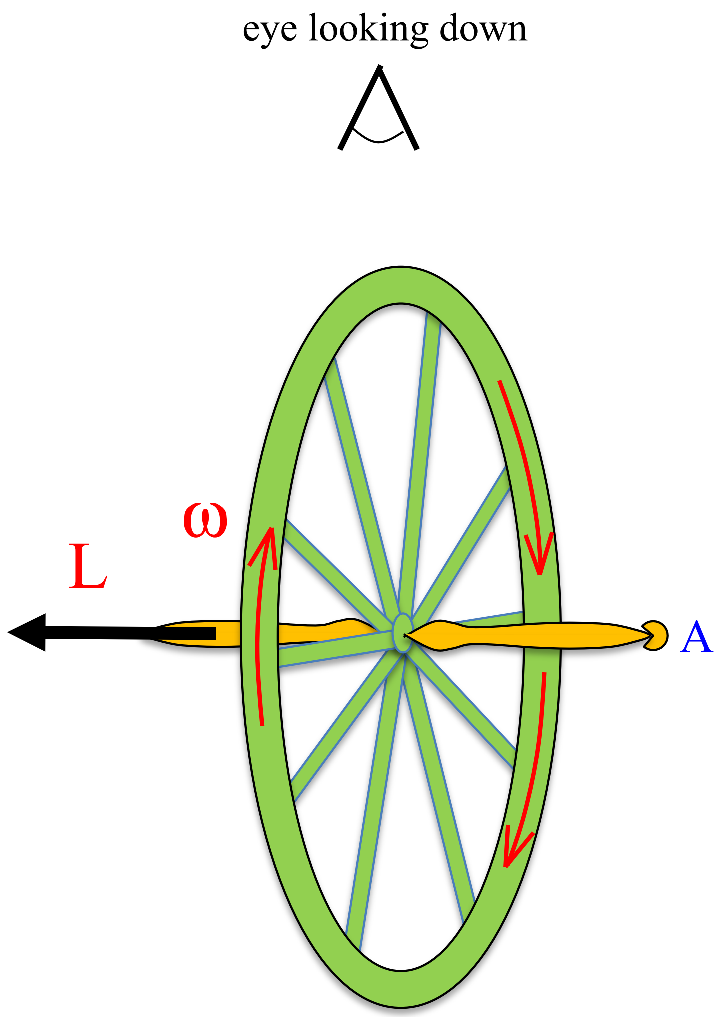


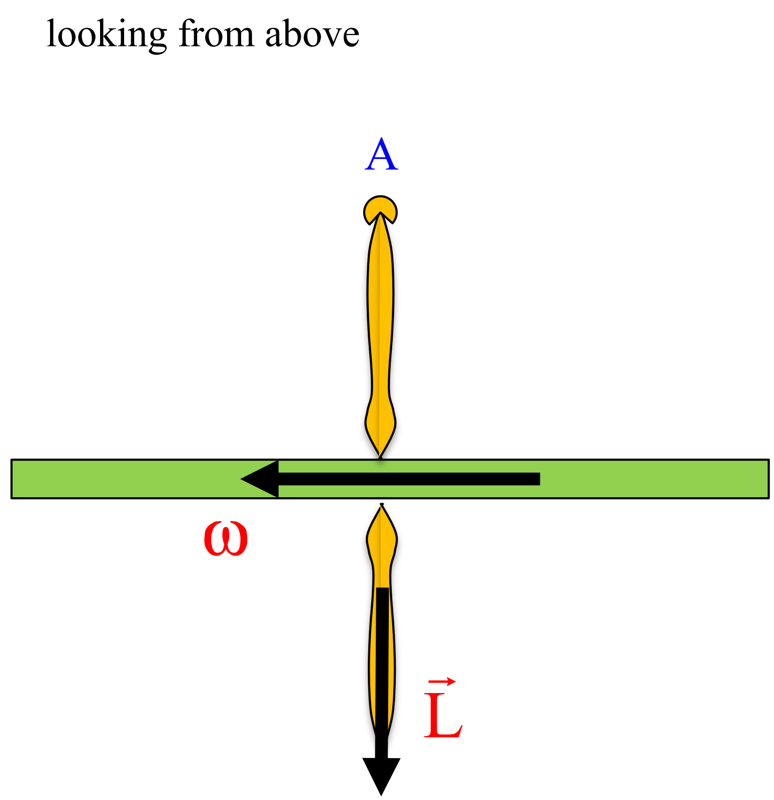
The torque produced by gravity is at right angles to the plane defined by $\vec{F}_g$ and $\vec{r}$.

With the torque (i.e., $\Delta\vec{L}$) at right angle to $\vec{L}$, the direction of the change of angular momentum demands that the body's axle must precess!

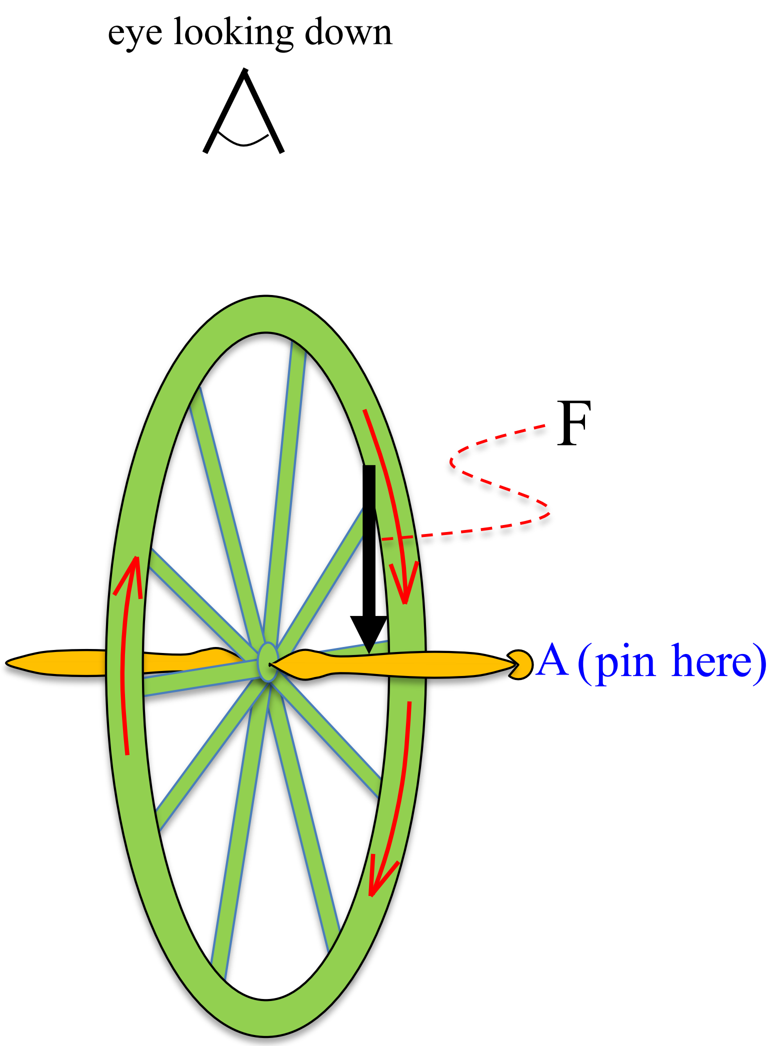


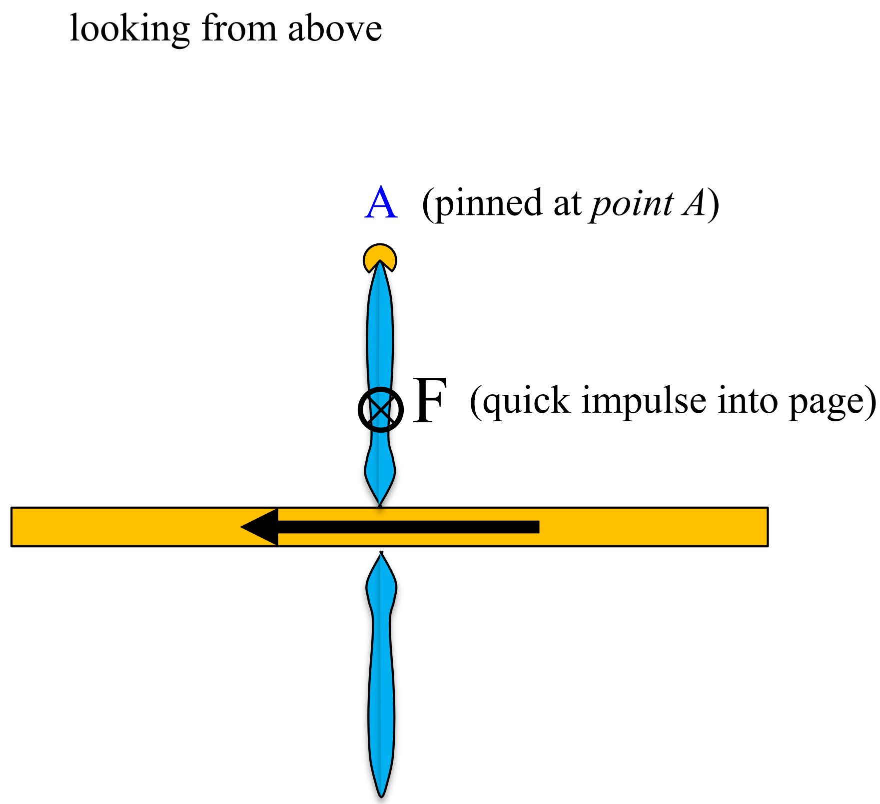
--What about the jerking wheel? Assume a clockwise rotation. The angular velocity (and, hence, angular momentum) vector is shown below in the sketch with the view from above shown to the right (I've put a nub at *point A* for reference).



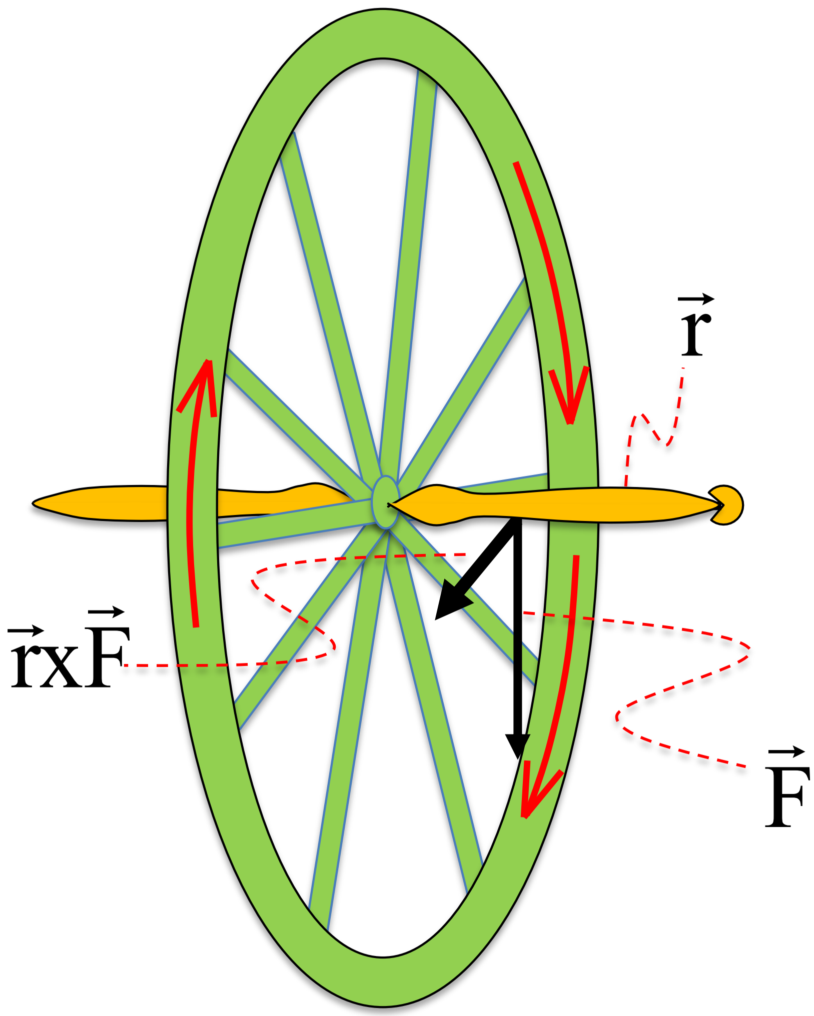


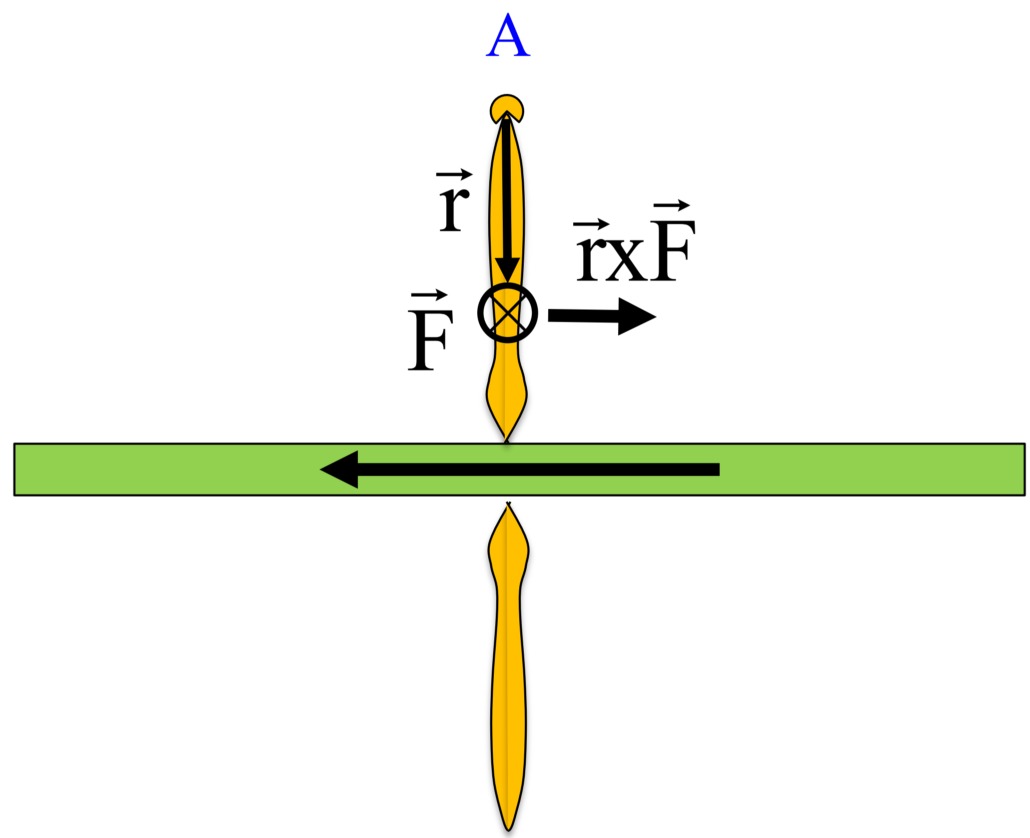
--Let's try to rotate the wheel about *point A* by applying a quick downward force F to *point A*.



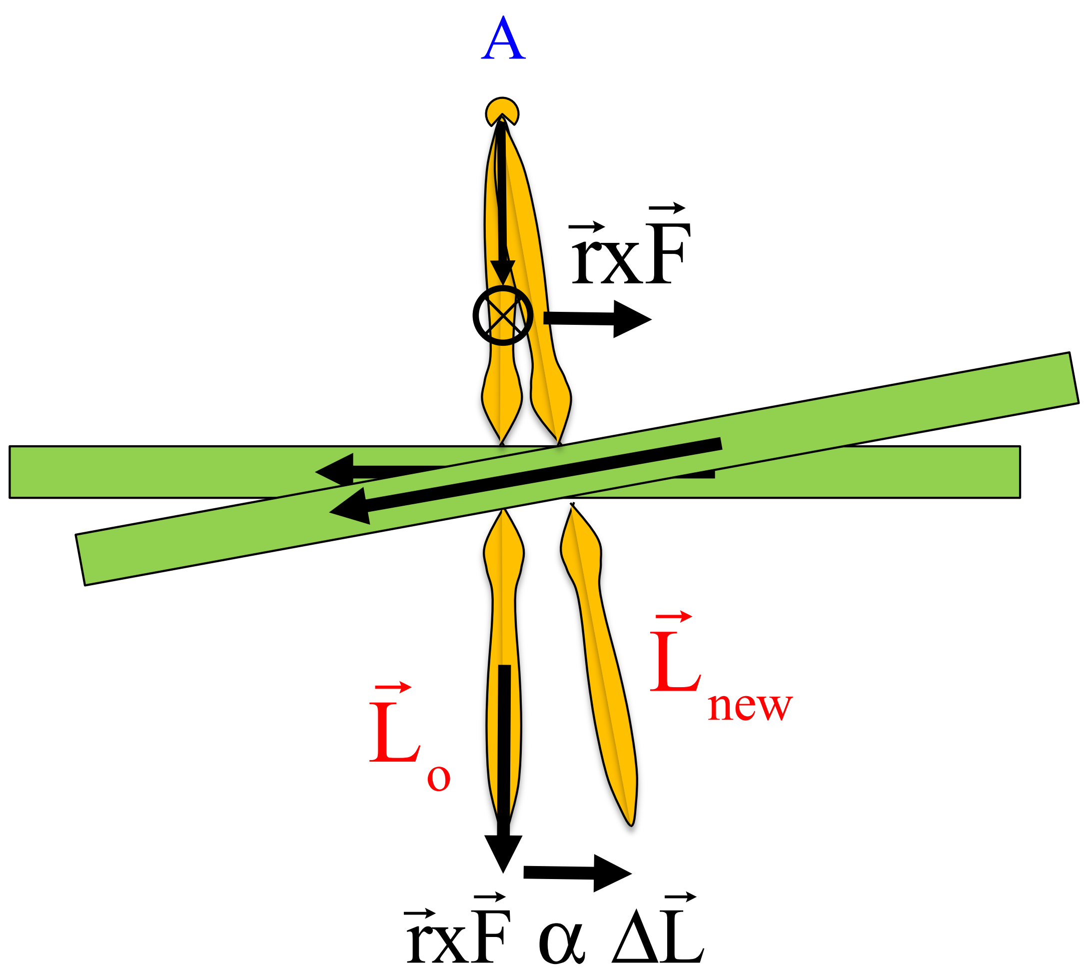


--Note that the direction of the torque applied by $\vec{F}$ is NOT in the direction of the angular momentum vector, which is along the axle.





--If the torque is in the direction of the CHANGE OF angular momentum, then the NEW angular momentum direction must be as shown below . . . and hence we predict precession!



$$(\text{as } \vec{\tau} = \frac{\Delta\vec{L}}{\Delta t})$$

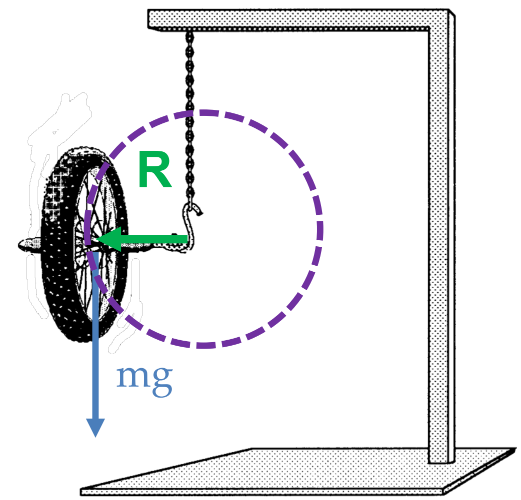
Coming full circle (see what I did there?)

- On the first day of this unit, we saw THE WHEEL.
- What happened when it was released while at rest?

  Gravity exerted a torque and it rotated down to a horizontal position.

- What happened when it was released while spinning?

  It stayed mostly vertical and precessed about a horizontal circle – not witchcraft, but physics!

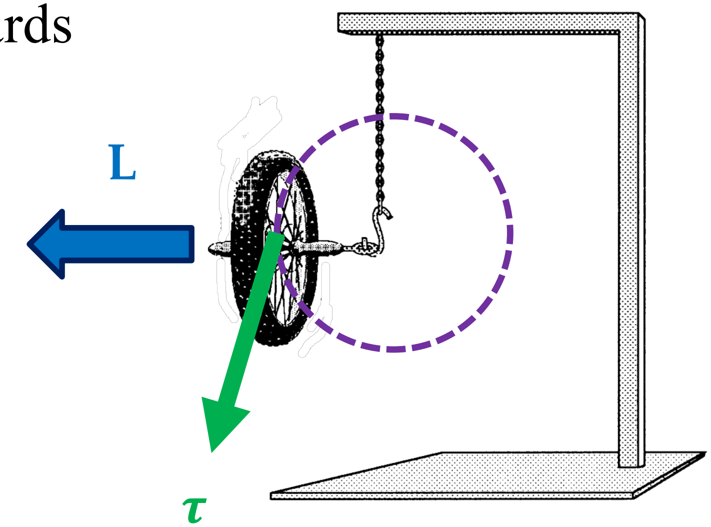


This is all about vectors and how they interact. Let's look at the wheel from two different views…

The Wheel – Explained!

Using the right hand rule, we know that the **torque** exerted by gravity is **pointing out of the page** towards us right now.

Also using the right hand rule, we can find the **angular momentum vector** as the wheel rotates. For now, let's say the wheel is rotating into the page, so its angular momentum vector is to the **left**.

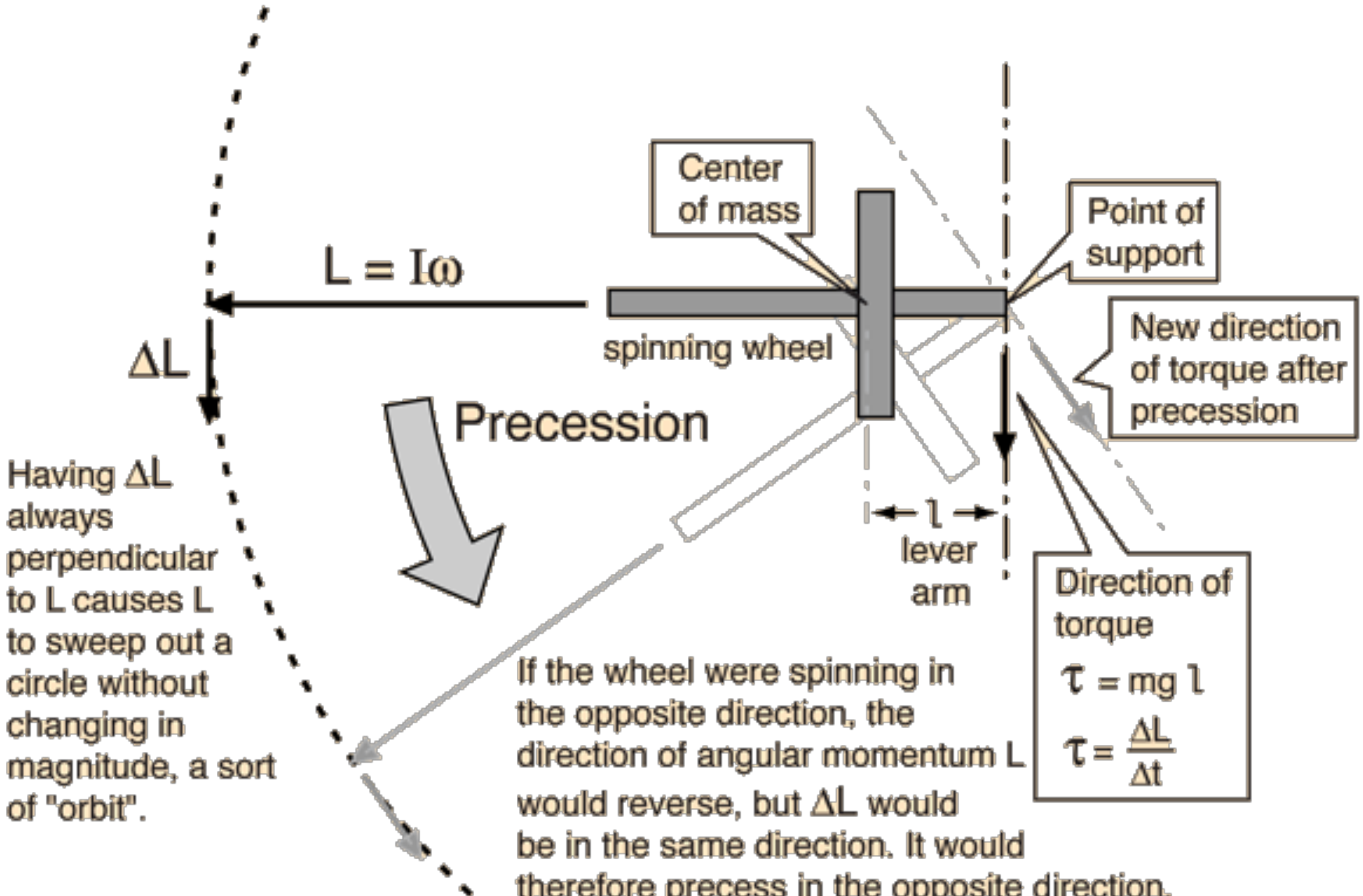


These two vectors are at right angles to each other. Previously, we saw that a force acting at a right angle to a velocity vector caused the velocity to change *direction* but not *magnitude*. When did we see this? (circles!)

Therefore, the torque vector won't change the magnitude of the angular momentum vector, but will change its direction. The wheel will move towards the direction of the torque…at which point the torque is pointing in a slightly different direction, so the wheel continues to move!

The Wheel – Explained!

In another view of the same thing, looking down from the top on the situation:

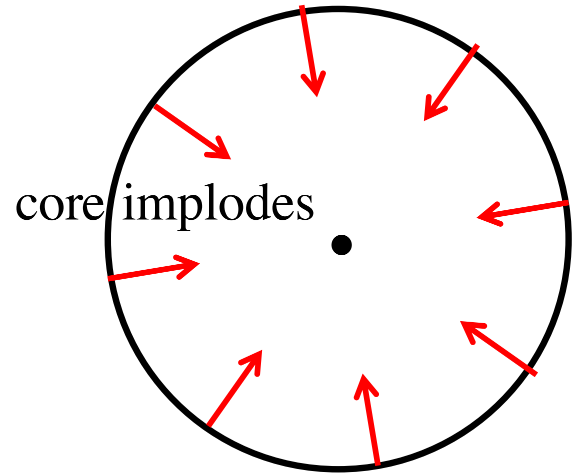


http://hyperphysics.phy-astr.gsu.edu/hbase/rotv2.html

A good video showing this…

- From Veritasium (an awesome physics channel):
  - Gyroscopic precession
  - "Anti-gravity" wheel

Example 6: In 1967 as a graduate student, Jocelyn Bell (aca Dame Jocelyn Bell Burnett) observed, in the face of scant support from her advisor, Antony Hewish, the first pulsar. In 1974, in a classic "keep 'em barefoot and pregnant" move, the all male, presumably all white Nobel committee gave Hewish the Nobel Prize in Physics for the discovery while ignoring Bell altogether. With that monumental injustice in mind, consider the lowly pulsars:



When a star with a core between 1.4 and 1.8 solar masses dies, it explodes spectacularly in what is called a supernova. (Example: In 1054, a supernova occurred that was observed by the Chinese and was visible *during the day* for two weeks.) When a supernova happens, the outer part of the star blows outward creating what is called a nebulae (the supernova in 1054 created the *Crab Nebulae*) and the core is blown inward. The *implosion* is so violent that it forces electrons into the nuclei of their atoms (removing all the space in the atoms in the process) where they combine with the protons there to produce neutrons that stop the implosion by literally jamming up against one another. With all that space removed, the resulting structure is incredibly dense (think *a thousand Nimitz class aircraft carriers* compressed into the size of a marble) and small (think 10 to 15 kilometers across).

(con't) The significance of all of this is that nature provides us with a WICKED example of *conservation of angular momentum*.

How so? There are no external torques acting during the supernova, so angular momentum is conserved. The enormously massive structure spread out over hundreds of thousands of kilometers starts out with a HUGE RADIUS and *angular momentum* even though its *angular speed* is low (the sun takes 25 days to rotate once about its axis). In other words, its *angular momentum* looks like:

$$L = I_{before} \qquad \omega_{before}$$

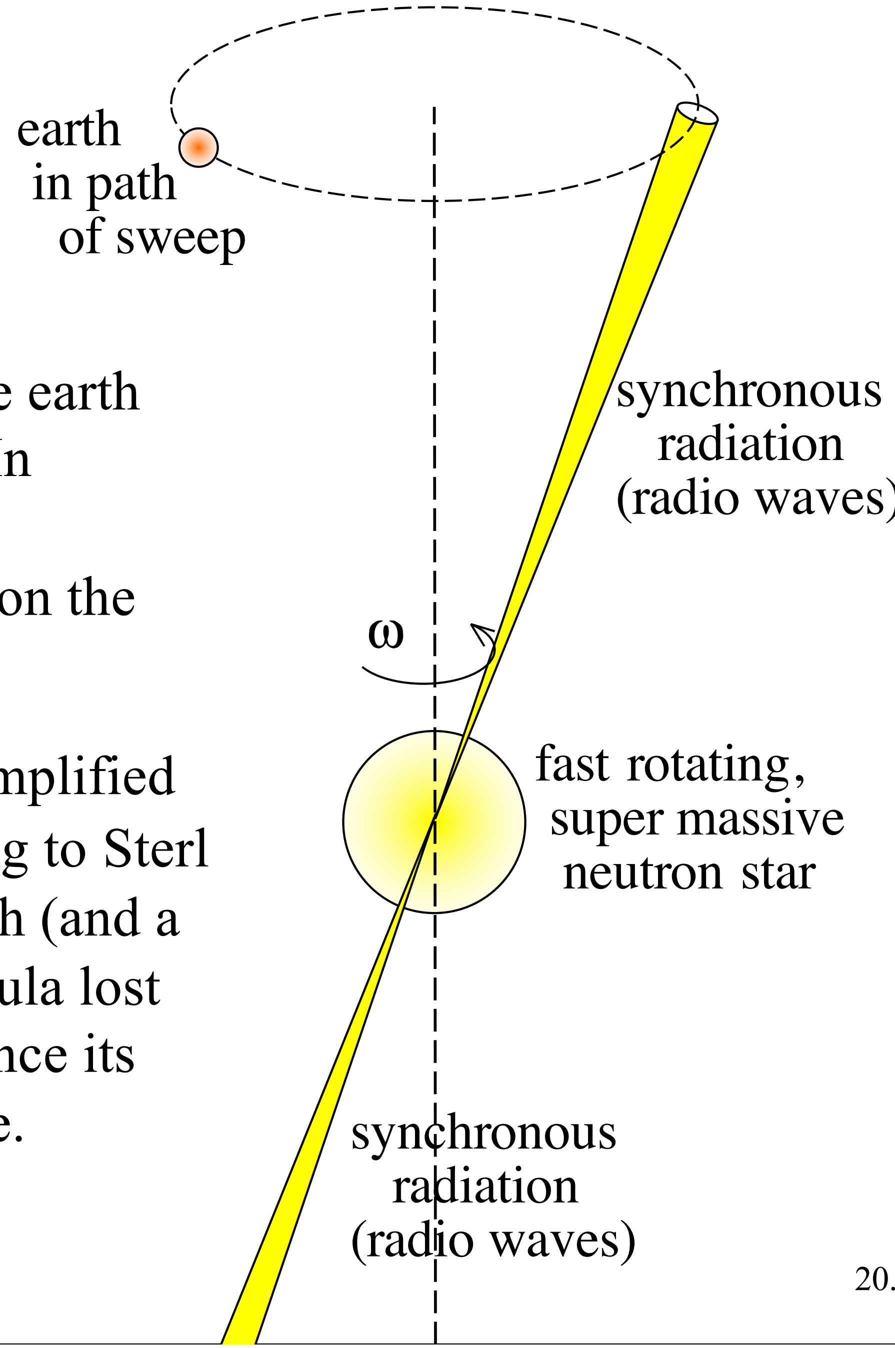
After the supernova, the *moment of inertial* drops precipitously because the radius goes from *several hundred thousand kilometers* to, maybe, *10 kilometers* during the explosion, BUT THE ANGULAR MOMENTUM STAYS THE SAME which means the angular velocity skyrockets. In other words, the *final* angular momentum relationship will look like:

$$L = I_{after}\,\omega_{after}$$

In short, pulsars (neutron stars) are super dense structures that rotate anywhere from a *few cycles per second* all the way up the *several hundred cycles per second*, all as a consequence of *conservation of angular momentum*.

But what's really cool is that they put out what is called *synchronous radiation*—radiation that is very directional and that is in the radio frequency range. So if the sweep of radiation of one of these fast rotating objects just happens to cross the earth's path, a blast of radio wave will hit the earth *every time the star completes one rotation*. In other words, we can *hear* them using a radio telescope. This is what you will experience on the next slide. Pretty amazing!

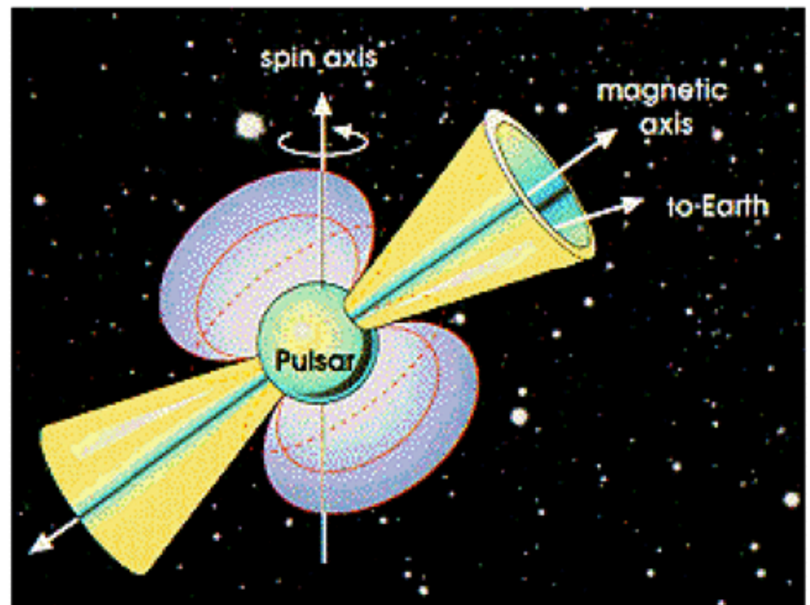
And as a small side-point, I've REALLY simplified what's going on with these things. According to Sterl Phinney, Professor of Astrophysics at Caltech (and a Poly parent), the progenitor of the Crab Nebula lost 99% of its angular momentum during and since its supernova. More about this on the next slide.



Pulsars

- Another cool application of angular momentum is **pulsars**
  - A **pulsar** is what's left over after a star collapses and goes nova
  - Normally, a star is <u>huge</u> (sun-like) and rotates at some (relatively small) angular velocity. When that star collapses, its mass compresses into area only a few km across (think city-sized) → $I\Downarrow$ so $\boldsymbol{\omega}\Uparrow$
    - This releases a lot of energy (the "supernova" explosion)

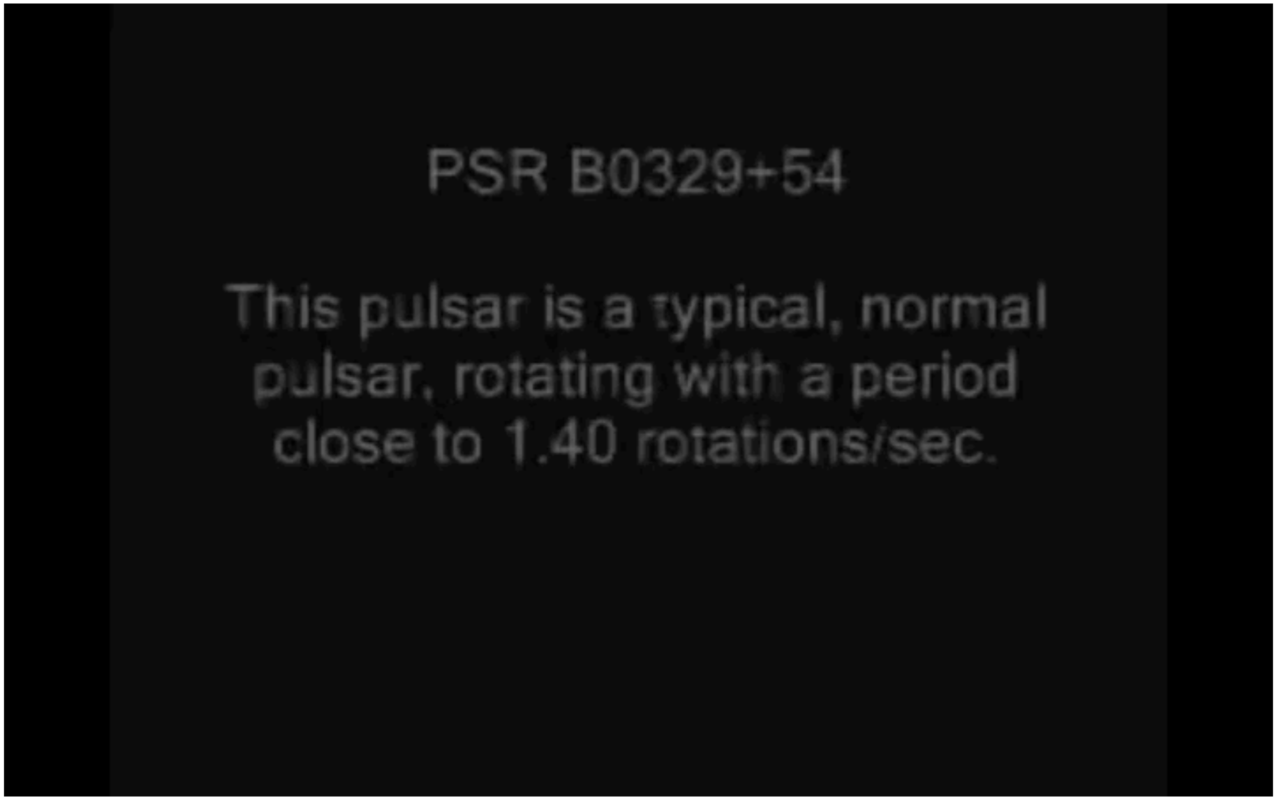
- The dense core left over is spinning more quickly, and what we pick up in our astronomical equipment is a "pulsing" signal (hence the name)
- This happens because the magnetic field of the star isn't aligned with its rotation axis.



https://www.space.com/32661-pulsars.html

Listening to pulsars

- We can "hear" this signal by converting the radio signals from pulsars into audible "clicks" with the same frequency, like this!



GOALLESS PROBLEMS

The following is a very brief summary of the things you need to consider when doing goalless problems. Note that you will be working in groups of two or three, your problem will be worth 20 test points and each group will get the grade. Once you've been given your problem:

--the underlying theme should be, "what can I find out about this situation?"

--identify what you know;

--take a few minutes to map out what you might determine and how you might do that;

--try to use concepts from <u>all</u> of the units we've done this year (not rotation);

--use more than one approach to determine a quantity if possible—box results;

--make drawings (f.b.d.'s, etc.) to make it clear what you are doing;

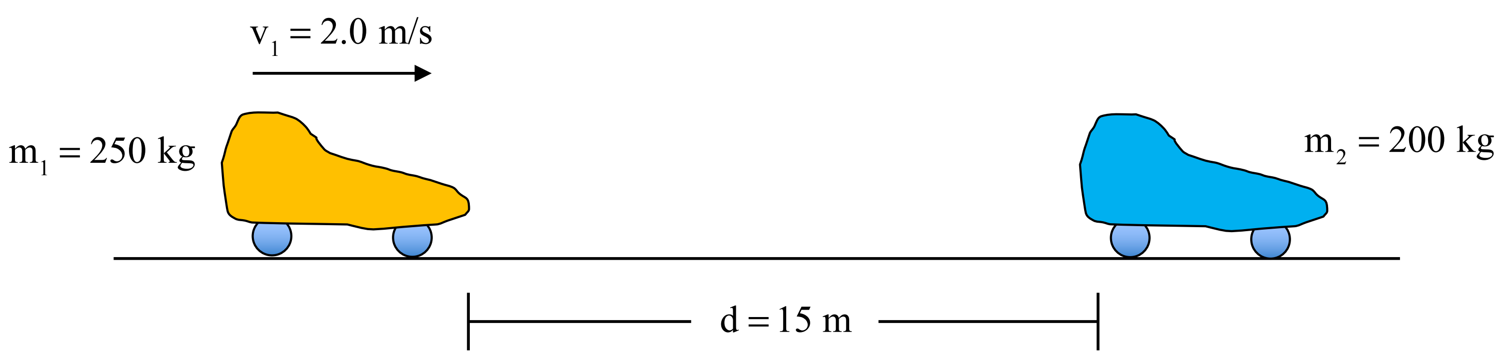
--BLURB your brains out—everything you do should be preamble with a blurb;

--visually display results when possible (use graphs);

GOALLESS PROBLEMS

Approach the following goalless problem as you would on the Block Day. To add a bit of zest to the proceedings, I will give a few extra credit points to any group whose total number of determined unknowns is within 10 of the number I come up with . . .

Several students are riding in bumper carts at an amusement park. The combined mass of cart A and its occupants is 250 kg. The combined mass of cart B and its occupants is 200 kg. Cart A is 15 meters behind cart B and moving to the right at 2.0 m/s when the driver decides to bump into car B, which is at rest.



Cart A accelerates at 1.5 m/s/s to a maximum speed of 5.0 m/s, then continues at that constant velocity until it strikes car B.

After the collision, which lasts 0.3 seconds, cart B moves to the right at a speed of 4.8 m/s.

Shortly thereafter, cart B is just able to hold traction around a sharp, 3.0 meter left-hand turn.

For the Rotational Motion test!

1.) There will be multiple-choice questions from the posted questions on the class Website – they are split into two pdfs but questions will be drawn from both . As you will have had the chance to see all of the possible questions beforehand, these will be worth 3 pts each on the test. There may also be a few very short answer questions (e.g. yes/no statements about situations; quick "what's the rotational counterpart to…" type questions; general information questions).

2.) Know how to use the rotational kinematics equations. (I will provide these on the test so you don't have to memorize them.)

3.) Know how to relate a rotating body's angular velocity or angular acceleration to the translational velocity or acceleration of a point on the body. That is, understand how and $v = r\omega$ work. $a = r\alpha$

4.) Know what the moment of inertia tells you, what the definition of the moment of inertia for a group of discrete masses is, and what the moment of inertia for a point mass is. I will provide all other needed moment of inertia relationships on the test.

5.) Know how to use the parallel axis theorem.

SUMMARY OF WHAT TO KNOW!

6.) Know the three ways to calculate a torque (by name!) and how to draw the r vector.

7.) Be able to determine the DIRECTION of a rotating body's angular velocity vector. Know what the pieces of " $\omega = -(3 \text{ rad/sec})\hat{i}$ " tell you.

8.) Be familiar with the examples we've analyzed a lot with different methods (rotating beam pinned somewhere along its length, rolling ball down ramp, simple Atwood Machine, merry go round).

9.) I will pick, possibly, two of the problems that follow. I would suggest you work in teams to determine how to do each, then get together and talk about each. In any case, you should know the ins-and-outs of each problem.

NOTE: you should focus on problems 1, 2, 3, 4, 5, and 8. The others are interesting and a good brain teaser to see how well you really understand things, but the ones listed above are the ones we will choose from for the exam.

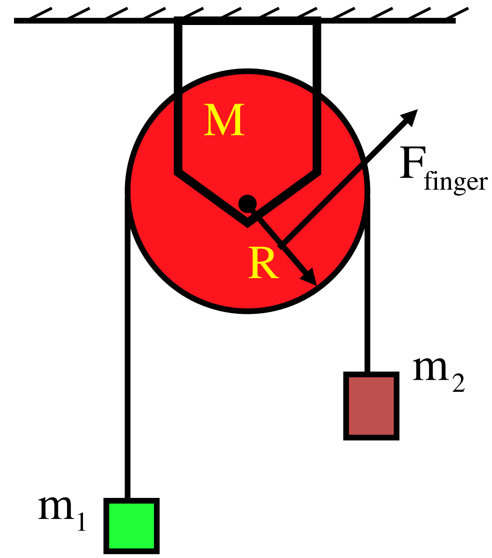
Another note: for each of these, make sure you understand how to derive equations (meaning start with a governing equation like N2L, conservation of energy, momentum, etc) and fit them to the situation (meaning put in the proper variables).

On the next several pages you will find synopses of the problems #1, 2, 3, 4, 5 and 8 (i.e., the problems alluded to on the previous slide above). The full blown problems *with solutions* can be found on the class Website. These problems do not include short answer or multiple choice questions, which you will run into on your block-day test, but they are indicative of a goodly part of your test you will take then.

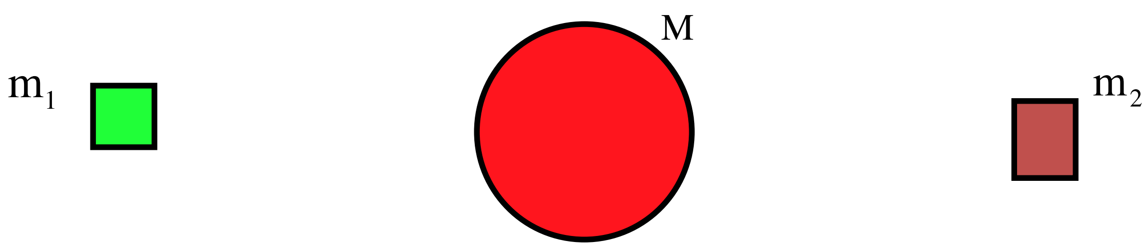
1.) A massive pulley is used in an Atwood machine. What is known is:

$$m_1,\ m_2,\ M, R, g, \text{ and } I_{cm,pully} = \frac{1}{2}MR^2$$

A finger is used to keep the pulley from rotating. Its force is applied to the pulley at "R/2" as shown in the sketch (you can assume the force is perpendicular to the radius vector).



a.) Draw a f.b.d. for both masses and the pulley.



b.) How much force must the finger apply to keep the system stationary?

c.) You remove the finger and the system begins to accelerate. What is the magnitude of the acceleration of the masses?

d.) What is the angular acceleration of the pulley (include the sign)?

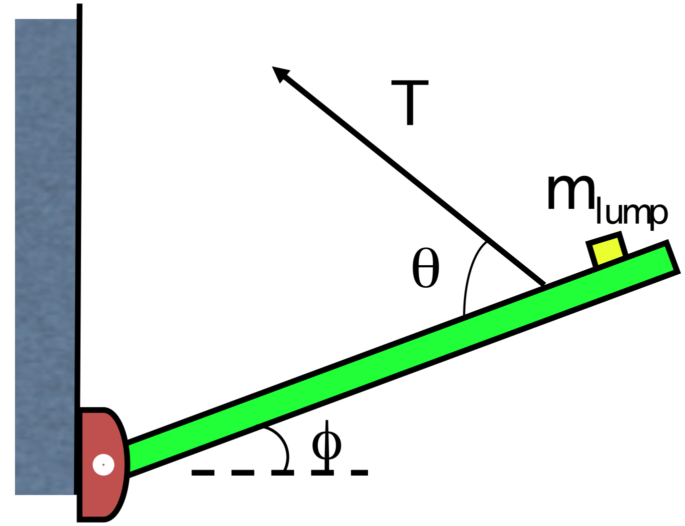
e.) The mass m_2 drops a distance "h." Once there, what is its velocity magnitude?

f.) For #e, what is the *angular velocity* of the pulley?

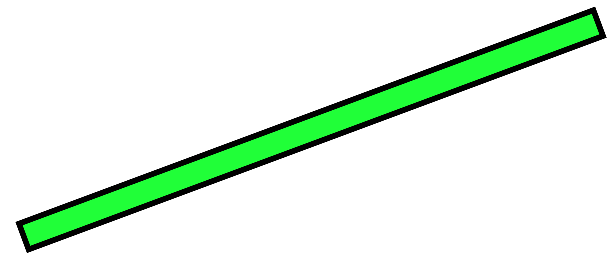
g.) For #e, what is the pulley's *angular momentum*?

2.) A beam of length "L" is pinned at an angle ϕ to a wall. Tension in a rope "3L/4" from the pin keeps it in equilibrium. There is amassive lump a distance "5L/6" units up the beam. What is known is:

$$m_{beam}, \; m_{lump}, \; L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_b L^2$$



a.) Draw a f.b.d. for the forces on the beam.



b.) What must the tension in the rope be for equilibrium?

c.) Use the Parallel Axis Theorem to determine the *moment of inertia* <u>of the beam</u> about the pin.

d.) Determine the *moment of inertia* of the lump, then system, about the pin.

e.) The rope is cut and the beam begins to angular accelerate downward. What is the beam's initial angular acceleration?

f.) What is the initial acceleration of the lump?

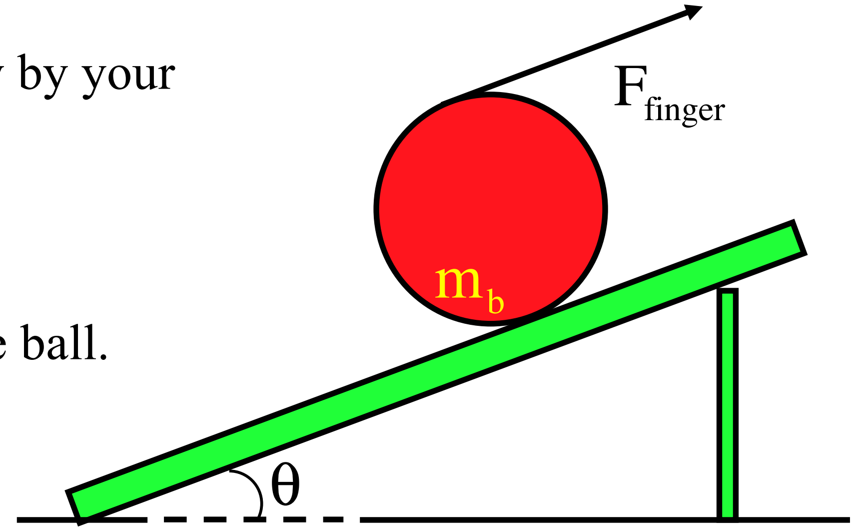
g.) The beam rotates about the pin. What is its angular velocity as it passes through the vertical?

h.) What is the beam's center-of-mass velocity as it passes through the bottom of the arc?

i.) What is the beam's angular momentum about the pin about that point?

3.) A thin skinned ball sits on an incline held stationary by your finger (ah, that finger again). What is known is :

$$m,\ R,\ g,\ \theta, \text{ and } I_{\text{cm of ball}} = \frac{2}{3}mR^2$$

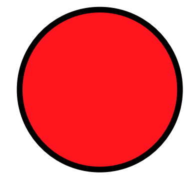


a.) Draw a f.b.d. identifying all the forces acting on the ball.

b.) Determine the magnitude of the finger force required to hold the ball in equilibrium.

c.) Use the Parallel Axis Theorem to determine the *moment of inertia* about the *point of contact* between the ball and the incline.

fbd



d.) With the finger removed, what is the magnitude of the acceleration of the ball's center of mass?

e.) What is the ball's angular acceleration about its center of mass?

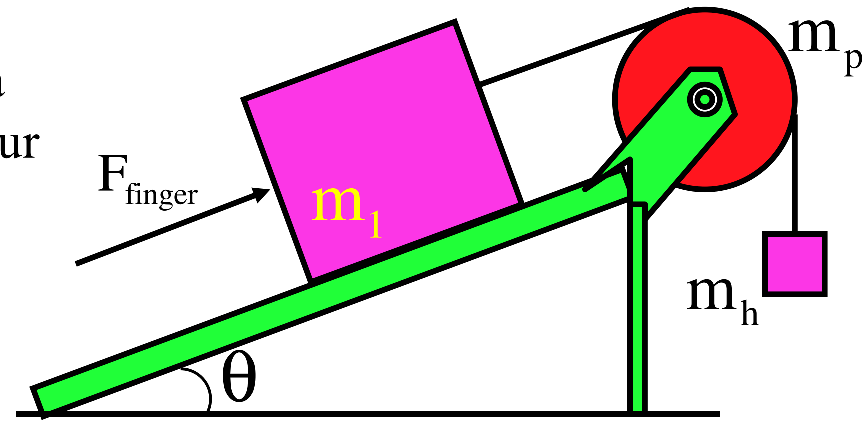
f.) The ball drops a distance "h" from rest. What is the magnitude of the velocity of its center of mass?

g.) After dropping "h," what is the ball's angular velocity?

h.) What is the angular momentum of the ball after dropping "h?"

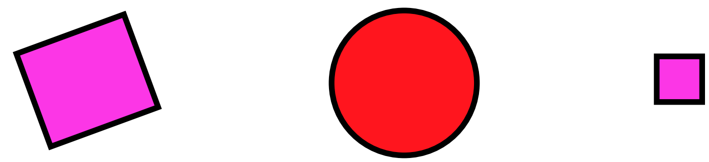
4.) A hanging mass is attached to a string which is threaded over a massive pulley and attached to a second mass sitting on a frictionless incline. Your finger keeps everything stationary. Known is:

$$m_1,\ m_h,\ m_p,\ R,\ g,\ \theta,\ \text{and}\ I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2$$



a.) Ignoring the forces acting at the pulley's pin, draw a f.b.d. identifying all the forces acting on both masses and the pulley.

b.) Determine the finger force required to just hold the system in equilibrium.



c.) The finger is removed and the system begins to accelerate. What is its acceleration magnitude? (Assume the acceleration is down the incline.)

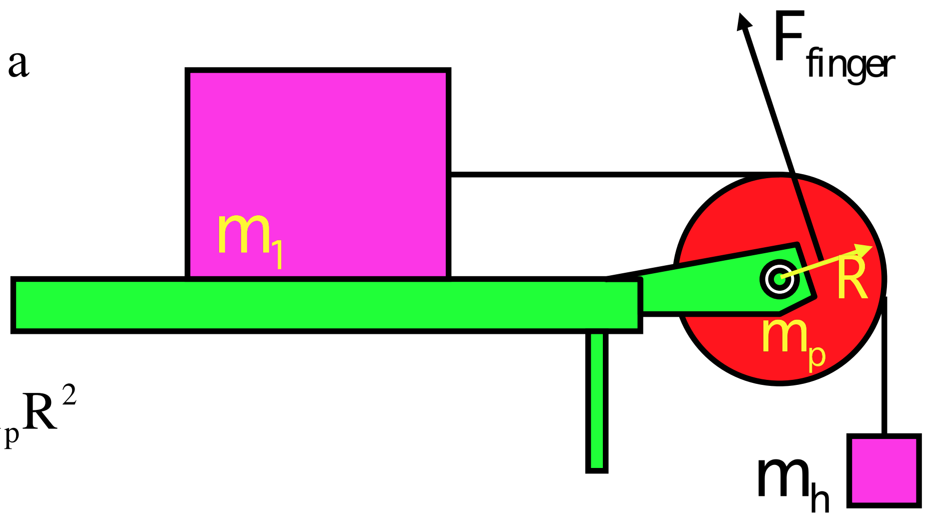
d.) What is the pulley's *angular acceleration*?

e.) The hanging mass drops from rest a distance "h." What is its velocity magnitude by the end of the drop?

f.) What is the angular velocity of the pulley at that point?

g.) What is the angular momentum of the pulley at that point?

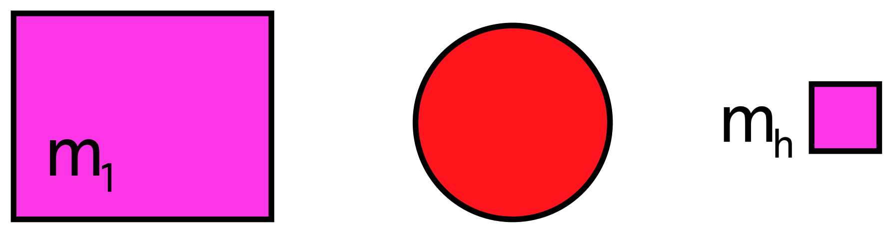
5.) A hanging mass is attached to a string which is threaded over a massive pulley and attached to a second mass sitting on an frictional tabletop. Your finger perpendicular to the radius vector and a distance R/3 from the axis of rotation maintains motionlessness. Known is:



$$m_1,\ m_h,\ m_p,\ R,\ g,\ \mu_k,\ \text{and } I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2$$

a.) Draw a f.b.d. identifying all the forces acting on both masses and the pulley.

b.) Determine the force required of the finger to keep the system in equilibrium.



c.) The finger is removed and the system begins to accelerate. What is the hanging mass's acceleration magnitude?

d.) What is the pulley's *angular acceleration*?

e.) The hanging mass drops a distance "h." What is its velocity magnitude at that point?

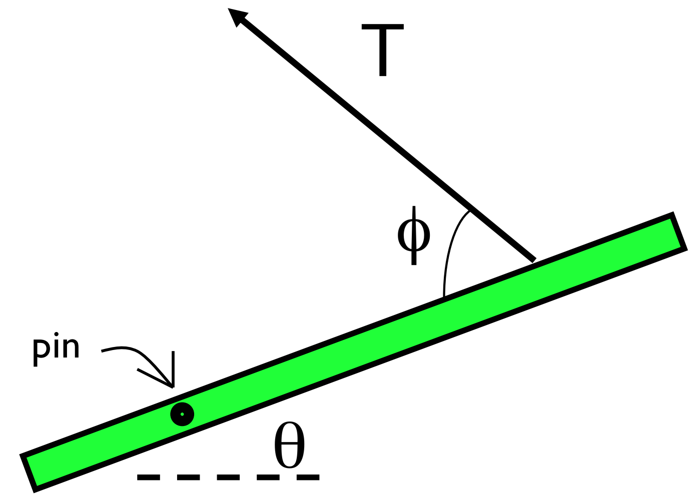
f.) What is the pulley's *angular velocity* at that point alluded to in Part e?

g.) What is the pulley's *angular momentum* at that point alluded to in Part e?

I.)

8.) A beam of length "L" is pinned at an angle θ a quarter of the way up the beam (i.e., at L/4). Tension in a rope three-quarters of the way from the end keeps it stationary. What is known is:

$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$



All the same questions asked in Question #2!